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On the stability of polynomial eigenvalue problems solved via linearization

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In this work, we investigate the accuracy and stability of polynomial eigenvalue problems expressed in the Lagrange basis that are solved by linearization [1]. For the scalar case, it has been demonstrated that computing the roots of polynomials via the eigenvalues of a certain arrowhead linearization is backward stable under certain conditions [2]. We extend this analysis to polynomial eigenvalue problems, and show the conditions under which the eigenvalues are computed with small backward errors. We also investigate a generalization of the arrowhead linearization to cover rational and nonlinear eigenvalue problems. We generate linearizations directly from sample values of the underlying problem at distinct nodes, avoiding the often ill-conditioned transformations between different bases. For certain special choices of nodes (real or on the unit circle), we can efficiently reduce the linearizations to block Hessenberg form [3, 4]. The algorithm simultaneously transforms all of the sample values of the polynomial matrix to the coefficients of orthogonal polynomials with respect to a discrete inner product that is based on the interpolation nodes.

References

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