Structured backward error analysis of polynomial eigenvalue problems solved by linearization

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Abstract

One of the most frequently used techniques to solve polynomial eigenvalue problems is linearization, in which the polynomial eigenvalue problem is turned into an equivalent linear eigenvalue problem with the same eigenvalues, and with easily recoverable eigenvectors. The eigenvalues and eigenvectors of the linearization are usually computed using a backward stable solver such as the QZ algorithm. Such backward stable algorithms ensure that the computed eigenvalues and eigenvectors of the linearization are exactly those of a nearby linear pencil, where the perturbations are bounded in terms of the machine precision and the norms of the matrices defining the linearization. With respect to the linearization, we may have solved a nearby problem, but we would also like to know if our computed solution is the exact solution of a nearby polynomial eigenvalue problem.

We perform a structured backward error analysis of polynomial eigenvalue problems solved via linearization. Through the use of dual minimal bases, we unify the construction of strong linearizations for many different polynomial bases. By inspecting the prototypical linearizations for polynomials expressed in a number of classical bases, we are able to identify a small number of driving factors involved in the growth of the backward error. One of the primary factors is found to be the norm of the block vector of coefficients of the polynomial, which is consistent with the current literature. We derive upper bounds for the backward errors for specific linearizations, and these are shown to be reasonable estimates for the computed backward errors.

Keywords: linearization, backward error, stability, dual minimal bases, strong linearization.