From Fiedler factorizations to block-Kronecker pencils

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Abstract

This seminar explores some recent advancements in the linearization of matrix polynomials. It traces the genealogy and connections between a number of recent developments in the field, which ultimately led to the development of a new family of linearizations of matrix polynomials. Motivated by the factorizations introduced by Fiedler [1] and in an effort to reduce the complexity of the representation of these pencils, it was discovered that the Fiedler factorizations form a subset of a much larger class of linearizations. This unifying framework, which we call "block-Kronecker pencils" [2], enables the linearization of a matrix polynomial by a block-anti-triangular pencil containing three distinct blocks. Two of the blocks encapsulate the recurrence relations of the underlying basis (in this case, the monomial one), and the third block contains the coefficients of the matrix polynomial. This seminar presents the underlying building blocks of this framework, namely, dual minimal bases and unimodular embeddings, and illustrates how these elements can be leveraged to develop a structure that is extremely useful in capturing a very wide class of linearizations of matrix polynomials. Furthermore, it discusses how these block-Kronecker pencils can be extended to other bases in a uniform fashion and applied to newly developing areas of research, such as the ℓ-ification of matrix polynomials.

References

[1] Fiedler, M., 2003. *A note on companion matrices.* Linear Algebra and its Applications, 372, pp.325-331.

[2] Dopico, F.M., Lawrence, P.W., Pérez, J., and Van Dooren, P., 2018. *Block Kronecker linearizations of matrix polynomials and their backward errors.* Numerische Mathematik, 140(2), pp.373-426.